New stochastic approach to the renormalization of the supersymmetric ϕ^4 with ultrametric.

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Abstract. We present a new real space renormalization-group map, on the space of probabilities, to study the renormalization of the SUSY ϕ^4. In our approach we use the random walk representation on a lattice labeled by an ultrametric space. Our method can be extended to any ϕ^n. New stochastic meaning is given to the parameters involved in the flow of the map and results are provided.

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1 Introduction

In this paper we use a new real space renormalization-group map to study renormalization of SUSY ϕ^4 theories. Symanzik's [1] work proved that ϕ^4 theories can be represented as weakly self-avoiding random walk models. They are in the same universality class as the self-avoiding random walk [2].

Our renormalization-group transformation is carried out on the space of probabilities. Real space renormalization-group methods have proved to be useful in the study of a wide class of phenomena and are used here to provide a new stochastic meaning to the parameters involved in the flow of the interacting constant and the mass, as well as the β function for SUSY ϕ^4.

The hierarchical models introduced by Dyson [3] have the feature of having a simple renormalization-group transformation. We use a hierarchical lattice where the points are labeled by elements of a countable, abelian group G with ultrametric δ; i.e. the metric space (G, δ) is hierarchical. The hierarchical structure of this metric space induces a renormalization-group map that is "local"; i.e. instead of studying the space of random functions on the whole lattice, we can descend to the study of random functions on L-blocks (cosets of G) [4]. Our method provides a probabilistic meaning to every parameter appearing in the flow of interaction constant and the mass for any ϕ^n.

This paper is organized as follows; in Section 2 we present the lattice and the corresponding class of Lévy processes which are studied here. In Section 3 we define the renormalization-group map and apply this to SUSY ϕ^4, in random walk representation. In Section 4 we use the results obtained in previous Section to give new ligth into the stochastic meaning of the map and results are presented.

2 The lattice with ultrametric and the Lévy process.

The hierarchical lattice used in this paper was introduced by Brydges, Evans and Imbrie [4].Here, we present a slight variant. Fix an integer $L \geq 2$. The points of the lattice are labeled by elements of the countable, abelian group $G = \oplus_{k=0}^{\infty} \mathbf{Z}_{L^d}$, d being the dimension of the lattice. A one-dimensional example can be found in Brydges et al [4] and a two dimensional example in Rodríguez-Romo [5]. An element X_i in G is an infinite sequence

$$X_i \equiv (..., y_k, ..., y_2, y_1, y_0) \;\; ; \;\; y_i \in \mathbf{Z}_{L^d} \quad \text{thus} \quad X_i \in G = \oplus_{k=0}^{\infty} \mathbf{Z}_{L^d},$$

where only finitely many y_i are non-zero.

Let us define subgroups

$$\{0\} = G_0 \subset G_1 \subset ... \subset G \quad \text{where} \quad G_k = \{X_i \in G | y_i = 0, i \geq k\} \quad (1)$$

and the norm $|\cdot|$ as

$$|X_i| = \left\{ \begin{array}{cc} 0 & \text{if } X_i = 0 \\ L^p & \text{where } p = \inf\{k | X_i \in G_k\} \text{ if } X_i \neq 0 \end{array} \right. \quad (2)$$

Then, the map $\delta : (X_{i+1}, X_i) \to |X_{i+1} - X_i|$ defines a metric on G. In this metric the subgroups G_k are balls $|X_i| \leq L^k$ containing L^{dk} points. Here the operation $+$ (hence - as well) is defined componentwise.

The metric defined by eq(2) satisfies a stronger condition than the triangle inequality, i.e.

$$|X_i + X_{i+1}| \leq \text{ Max}(|X_i|, |X_{i+1}|). \quad (3)$$

From eq(3), it is clear that the metric introduced is an ultrametric.

For the purposes of this paper we introduce the Lévy process as a continuous time random walk w. This is the following ordered sequence of sites in G;

$$(w(t_0), ..., w(t_0 + ... + t_n)) \;\;, w(t_0 + ... + t_i) = X_i \in G, \;\; T = \sum_{i=0}^{n} t_i, \; n \geq 0 \;\; (4)$$

where t_i is the time spent in $X_i \in G$ (waiting time at X_i) and T, fixed at this point, is the runnig time for the process. For convenience we take $X_0 = 0$.

We are not dealing with nearest neighbour random walks on the lattice, provided we mean neighbourhood with respect to the ultrametric distance δ previously defined. We propose the Lévy process we are dealing with, having a probability $P(w) = r^n e^{-rT} \prod_{i=0}^{n-1} q(X_{i+1}, X_i)$. Namely the continuous time random walk has a probability rdt (r is the jumping rate) of making a step in time (t, dt) and, given that it jumps, the probability of jumping from X_i to X_{i+1} is $q(X_{i+1}, X_i)$, conditioned to a fixed running time T for the process. $q(X_{i+1}, X_i)$ is an open function of the initial and final sites of jumps in the lattice G. Here we define Dw by $\int (\cdot) Dw =$
$\sum_n \sum_{[X_i]_{i=0}^n} \int_0^T \prod_{i=0}^n dt_i \delta(\sum_{i=0}^n t_i - T)(\cdot)$. From this follows $\int P(w) Dw = 1$.

Let the space of simple random walks of length n, be Λ_n, with probability measure $P(w)$, we construct on this space the weakly SARW model that represents SUSY ϕ^4 (through McKane-Parisi-Sourlas theorem [6]). We take advantage of this feature to provide a better understanding of SUSY ϕ^4 renormalization in terms of stochastic processes. This method can be straightforward generalized to any SUSY ϕ^n with ultrametric.

3   The renormalization-group map on the random walk representation of SUSY ϕ^4.

We propose a renormalization-group map on the lattice $R(X_i) = LX_i'$ where $X_i \in G$ and $LX_i' \in G' = G/G_1 \sim G$; i.e. the renormalized lattice G' is isomorphic to the original lattice G. Here $LX_i' = (..., y_2, y_1)$.

Besides we propose the action of the renormalization-group map on the space of random walks $R(w) = w'$, from w above as defined, to w'. Here, w' is the following ordered sequence of sites in $G' = G/G_1 \approx G$;

$$(w'(t_0'), ..., w'(t_0' + ... + t_k')) \ , \ \text{where} \tag{5}$$

$$w'(t'_0, ..., t'_{i'}) = X'_{i'} \in G, \quad T' = \sum_{i'=0}^{k} t'_i, \ 0 \leq k \leq n, \ T = L^\varphi T'.$$

R maps $w(t_0)+G_1, w(t_0+t_1)+G_1, ..., w(t_0+...+t_n)+G_1$ to cosets $Lw(t_0), Lw(t_0+t_1),...,Lw(t_0 + ... + t_n)$ respectively. If two or more successive cosets in the image are the same, they are listed only as one site in $w'(t'_0), ..., w(t'_0+...+t'_k)$, and the times t'_j are sums of the corresponding t_i for which successive cosets are the same, rescaled by L^φ. For $\varphi = 2$, we are dealing with normal diffusion (this is the standard version of SUSY ϕ^4), in case $\varphi < 2$ with superdiffusion, and subdiffusion for $\varphi > 2$. In the following this parameter is arbitrary, so we can study general cases.

We can now work out probability measures at the $(p + 1)^{th}$ stage in the renormalization provided only that we know the probabilities at the pth stage. We integrate the probabilities of all the paths $w^{(p)}$ consistent with a fixed path $w^{(p+1)}$ in accordance with the following. Let $R(w) = w'$ be the renormalization-group map above as stated, then $P'(w')=$
$L^{\varphi k} \int Dw P(w) \chi(R(w) = w')$. Here $R(w) = w'$ is a renormalization-group transformation that maps an environment $P(w)$ to a new one, $P'(w')$, thereby implementing the scaling.

Hereafter

$$m_{j'} = \sum_{i'=0}^{j'} n_{i'} + j' \quad \text{and} \tag{6}$$

$$n = \sum_{i'=0}^{k} n_{i'} + k \qquad 0 \leq j' \leq k, \text{ being}$$

$n_{i'} = max\{i|w(t_0+...+t_j) \in LX_{i'}, \forall j \leq i\}$; i.e. the number of steps (for paths w) in the contracting G_1 coset that, once the renormalization-group map is applied, has the image $LX'_{i'}$.

Concretely $P'(w')$ can be written like

$$P'(w') = L^{\varphi k} \sum_{[n_{i'}]_{i'=0}^{k}} \sum_{[X_i]_{i=0}^{n}} \int \prod_{i=0}^{n} dt_i \prod_{j'=0}^{k} \delta(\sum_{i=m_{j'-1}+1}^{m_{j'}} t_i - L^\varphi t'_{j'}) \times \tag{7}$$

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$$\times \prod_{j'=0}^{k} \prod_{i=m_{j'-1}+1}^{m_{j'}} \chi(X_i \in LX'_{j'}\ )P(w).$$

It is straightforward to prove that the probability $P(w)$ where we substitute $q(X_{i+1}, X_i)$ by $c|X_{i+1} - X_i|^{-\alpha} \; \forall X_i, X_{i+1} \in G$ (c is a constant fixed up to normalization and α another constant), is a fixed point of the renormalization-group map provided $\varphi = \alpha - d$. Even more, if in $P(w)$ we substitute $q(X_{i+1}, X_i)$ by $c\left(|X_{i+1} - X_i|^{-\alpha} + |X_{i+1} - X_i|^{-\gamma}\right) \; \forall X_i, X_{i+1} \in G$, $\gamma >> \alpha$ (γ is an additional constant), then this flows to the very same fixed point of the renormalization-group map that in the first case. This holds provided $log\left(\frac{L^{-\alpha} - L^{-\gamma} - 2L^{d-\gamma-\alpha}}{L^{-\alpha} - 2L^{d-\gamma}}\right) \rightarrow$ 0 and $\varphi = \alpha - d$.

Let us substitute $q(X_{i+1}, X_i)$ by $q_1(|X_{i+1} - X_i|) + \epsilon b(X_{i+1}, X_i)$ where $q_1(|X_{i+1} - X_i|)$ is any function of the distance between X_{i+1} and X_i, both sites in the lattice; $b(X_{i+1}, X_i)$ is a random function and ϵ a small parameter. We can impose on $b(X_{i+1}, X_i)$ the following conditions.

a) $\sum_{X_{i+1}} b(X_{i+1}, X_i) = 0$.

b) Independence. We take $b(X_{i+1}, X_i)$ and $b(X'_{i+1}, X'_i)$ to be independent if $X_{i+1} \neq X'_i$.

c) Isotropy.

d) Weak randomness.

In this case, $P(w)$ is still a fixed point of the renormalization-group map provided

$$\sum_{(n_j)_0^k} \sum_{(X_i)_0^n} \prod_{i=0}^{n-1} b(X_{i+1}, X_i) \prod_{j=0}^{k} \prod_{i=m_{j-1}+1}^{m_j} \chi(X_i \in LX'_j) = \qquad (8)$$

$$\prod_{j=0}^{k} b(X'_{j+1}, X'_j) L^{-\varphi k} (b(1)(L^d - 1))^{n_j}$$

where $b(1) = \frac{1 - L^{-\varphi}}{L^d - 1}$ is the probability of jumping from one specific site to another specific site inside the G_1 coset.

One formal solution to eq(8) is the following

$$
b(X_{i+1}, X_i) = \begin{cases} \frac{1-L^\varphi}{L^d-1} & \text{if } |X_{i+1} - X_i| = L \\ \sum_t \begin{pmatrix} d + \varphi \\ t \end{pmatrix} f(X_{i+1})^t f(X_i)^{d+\varphi-t} & \text{where } |X_{i+1} - X_i| > L \end{cases}
$$

(9)

up to proper normalization. Here $f(X_i)$ and $f(X_{i+1})$ are homogeneous function of sites in the lattice, order -1. Besides they add to ..., 1) and are positive defined. Since in the limit $d + \varphi \to \infty$ (provided the mean remains finite) binomial probability distribution tends to Poissson distribution; we think a nontrivial SUSY ϕ^4 theory could be included in this case [7].

The random walk representation of the SUSY ϕ^4 is a weakly SARW that penalizes two-body interactions, this is a configurational measure model. Configurational measures are measures on Λ_n. Let $P_U(w)$ be the probability on this space such that

$$
P_U(w) = \frac{U(w)P(w)}{Z}
$$

(10)

where $Z = \int U(w)P(w)Dw$ and $P(w)$ is the probability above described, thus a fixed point of the renormalization-group map. Besides $U(w)$ is the energy of the walks. To study the effect of the renormalization-group map on $P_U(w)$ we need to follow the trajectory of $U(w)$ after applying several times the renormalization-group map.

Therefore, from previous definition of the renormalization-group map follows;

$$
P'_{U'}(w') = L^{\varphi k} \int P_U(w)\chi(R(w) = w')Dw
$$

where $Z' = Z$, thus

$$
U'(w') = \frac{\int Dw P(w)\chi(R(w) = w')U(w)}{\int Dw P(w)\chi(R(w) = w')}
$$

(11)

Note that eq(11) can be view as the conditional expected value of $U(w)$ given that the renormalization-group map is imposed. Therefore and hereafter, to simplify notation, we write eq(11) as $U'(w') = < U(w) >_{w'}$.

In the random walk representation of the SUSY ϕ^4 model with interaction λ, and mass (killing rate in the stochastic framework) m, U is as follows.

$$U(w) = \prod_{X \in G} e^{-m \sum_{i \in J_X} t_i - \lambda \sum_{i < j \in J_X} t_i t_j \mathbf{1}\left\{w(t_i) = w(t_j)\right\}}, \qquad (12)$$

being $m < 0$ and $\lambda \gtrless 0$ (small) constants. Here we set a randomly free running time for the process T. The probability $P_U(w)$, where $U(w)$ is defined as in eq(12), flows to a fixed form after the renormalization-group map is applied. This fixed form is characterized by the renormalized energy

$$U'(w') = \prod_{X' \in G} e^{-m' \sum_{i' \in J_{X'}} t'_{i'} - \lambda' \sum_{\substack{\{i',j'\} \in J_{X'}}}^{i' < j'} t'_{i'} t'_{j'} \mathbf{1}_{(w(t'_{i'}) = w(t'_{j'}))}} \times \qquad (13)$$

$$\left\{ 1 + \eta'_1 \sum_{\substack{i' < j' < k' \\ \{i',j',k'\} \in J_{X'}}} t'_{i'} t'_{j'} t'_{k'} \mathbf{1}_{(w(t'_{i'}) = w(t'_{j'}) = w(t'_{k'}))} + \right.$$

$$\left. + \eta'_2 \sum_{\substack{i' < j' \\ \{i',j'\} \in J_{X'}}} t'_{i'} t'_{j'} \mathbf{1}_{(w(t'_{i'}) = w(t'_{j'}))} + \eta'_3 \sum_{i' \in J_{X'}} t'_{i'} \right\} + r'.$$

Here

$$m' = L^\varphi m + m'_1 \quad where \qquad (14)$$

$$m'_1 = \gamma_1 \lambda - \gamma_2 \lambda^2 + r_{m'_1}. \qquad (15)$$

$$\lambda' = L^{2\varphi - d} \lambda - \chi \lambda^2 + r_{\lambda'}. \qquad (16)$$

$$\eta'_1 = \eta_1 L^{3\varphi - 2d} + \eta \lambda^2. \qquad (17)$$

$$\eta'_2 = \eta_1 A + L^{(2\varphi - d)} \eta_2 \quad and \qquad (18)$$

$$\eta'_3 = \eta_1 B + \eta_2 \gamma_1 + L^\varphi \eta_3. \qquad (19)$$

All parameters involved in eq(15), eq(16), eq(17), eq(18) and eq(19); namely γ_1, γ_2, χ, η, A and B have precise, well defined formulae [5]. They are linearized conditional expectations of events inside contracting G_1 cosets

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which, upon renormalization, maps to a fixed random walk with totally arbitrary topology. Even more, we have precise formulae for all the remainders, also [5]. Concretely speaking, γ_1 and γ_2 are contributions to renormalized local times coming from one and two two-body interactions inside the contracting G_1 cosets, respectively. χ is the two two-body interaction (inside the contracting G_1 cosets) contribution to renormalized two-body interaction. η is the contribution to renormalized three-body interaction coming from two two-body interactions. Finally A and B are the one three-body interaction (inside the contracting G_1 cosets) contribution to renormalized two-body interaction and local time, respectively.

In the SUSY representation of ϕ^4 we can say that γ_1 and γ_2 are first and second order contributions of SUSY ϕ^4 to renormalized SUSY mass; χ is the second order contribution of SUSY ϕ^4 to renormalized SUSY ϕ^4; η is the second order contribution (the first order contribution is null due to topological restriccions) of SUSY ϕ^4 to renormalized SUSY ϕ^6. Finally, A and B are first order contributions of SUSY ϕ^6 (already generated at this stage by previous renormalization stages) to renormalized SUSY ϕ^4 and mass, respectively.

Eq(13) is presented in terms of the product of two factors. The first one (exponential) involves only; a) trivial flow of mass and interacting constant b) $\lambda\phi^4$ contribution (inside contracting G_1 cosets) to renormalized mass and $\lambda'\phi^4$ up to leading order. The second factor involves mixed terms; namely $\lambda\phi^4$ and ϕ^6 contributions (inside the contracting G_1 cosets) to renormalized mass, ϕ^4 and ϕ^6. ϕ^6 terms come into the scheme because they are produced from $\lambda\phi^4$ due to the fixed topology of the continuous-time random walk on the hierarchical lattice. This arrangment allows us to distinguish the physically meaningful (leading order) magnitudes. From this, we analyze some results in next section.

We can choose either representation to obtain the final formulae for parameters and remainders. Here and in Rodríguez-Romo S. [5] we choose the one to provide new stochastic meaning to renormalizing SUSY field theories.

We claim that this result is the space-time renormalization-group trajectory, for the weakly SARW energy interaction studied by Brydges, Evans and Imbrie [4] provided $\varphi = 2$ and $d = 4$. In their paper the trajectory of a SUSY ϕ^4 was studied (recall that this can be understood in terms of intersection of random walks due to Mc Kane, Parisi, Sourlas theorem) from a SUSY field-theoretical version of the renormalization-group map, on almost the same hierarchical lattice we are studying here. We improve the model by providing exact expressions for λ and m for each step the renormalization-group is applied in the stochastic framework, among others.

To obtain eq(13) we have introduced an initial mass term m, $O(\lambda^2)$ (this allows a factorization whose errors are included in the remainder r', automatically). We use the Duplantier's hypotesis [8] and assume all divergences of the SUSY ϕ^4 with ultrametric as coming from the vertices or interactions per site of the lattice. This hypothesis has been proved to be correct in dimension 2 by means of conformal field theory. Then, a formal Taylor series expansion is applied which is analyzed for each particular topology in the renormalized field theory (this is done in random walk representation) per site of the new lattice. Putting everything together and by induction, we obtain the final result.

We can apply the very same method to study any SUSY Φ^n model on this ultrametric space.

4 Renormalized SUSY ϕ^4 with ultrametric. The stochastic approach.

To start with, we write the physically meaningful (leading order) part of eq(13); namely eq(14), eq(15) and eq(16) in parameter space. Let us define the following vector

$$\mathbf{H} = (m, \lambda) \tag{20}$$

Here we have approached up to the most probable events (first order in SUSY representation). The action of the renormalization-group map (RG)

is expressed as

$$\mathbf{H}' = R(\mathbf{H}) = (m', \lambda'). \tag{21}$$

The fixed points in our theory, (m_1^*, λ_1^*) and (m_2^*, λ_2^*) are as follows.

  a) The trivial $m_1^* = \lambda_1^* = 0$.

  b) $\lambda_2^* = \frac{L^{2\varphi - d} - 1}{\chi}$; $m_2^* = \frac{\gamma_1 (L^{2\varphi - d} - 1)}{\chi(1 - L^{\varphi})} - \frac{\gamma_2 (L^{2\varphi - d} - 1)^2}{\chi^2 (1 - L^{\varphi})}$.

The nontrivial fixed point involves a renormalized two-body interaction which is inverse to the conditional expectation of two two-body interactions that renormalizes to a two-body interaction inside the contracting G_1 cosets (χ) given that the RG map is applied. Meanwhile the renormalized mass in this point is given in terms of two ratios. The first one involves the ratio of conditional expectations of one two-body interaction that renormalizes to local times (γ_1) inside a contracting G_1 coset and χ. The second ratio involves the conditional expectation of two two-body interactions that renormalize to local times (γ_2) inside a contracting G_1 coset and χ^2. Both; λ_2^*, m_2^*, are independent of the scaling factor L for large lattices.

As we come infinitesimally close to a particular fixed point, (called this $\mathbf{H}^*$), the trajectory is given completely by the single matrix M (its eigenvalues and eigenvectors). Namely

$$M_{ij} = \left. \frac{\partial R_i(\mathbf{H})}{\partial H_i} \right|_{\mathbf{H} = \mathbf{H}^*} \tag{22}$$

From the random walk representation of SUSY ϕ^4 with ultrametric, up to the most probable event approach (leading order in SUSY representation), we obtain

$$M = \begin{pmatrix} L^{\varphi} & \gamma_1 - 2\gamma_2 \lambda^* \\ 0 & L^{2\varphi - d} - 2\chi\lambda^* \end{pmatrix}, \tag{23}$$

where λ^* can be either λ_1^* or λ_2^*.

The eigenvalues and eigenvectors of this matrix are as follows.

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a) $l_1 = L^\varphi$ with eigenvector $(m, 0)$.

b) $l_2 = L^{2\varphi - d} - 2\chi\lambda^*$ with eigenvector $\left(m, -\frac{L^\varphi - 2L^{2\varphi - d} + 2\chi\lambda^*}{\gamma_1 - 2\gamma_2\lambda^*}m\right)$, where λ^* can be either λ_1^* or λ_2^*.

For $L \geq 2$ and $\varphi > 0$; both fixed points (m_1^*, λ_1^*) and (m_2^*, λ_2^*) are repulsive in the direction of the eigenvector $(m, 0)$, marginal if $\varphi = 0$ and attractive if $\varphi < 0$. The trivial fixed point (m_1^*, λ_1^*) is repulsive in the direction of the eigenvector $\left(m, -\frac{L^\varphi - 2L^{2\varphi - d} + 2\chi}{\gamma_1}m\right)$ provided $\varphi > d/2$, marginal if $\varphi = d/2$ and attractive otherwise. Finally, the fixed point (m_2^*, λ_2^*) is repulsive in the direction of the eigenvector $\left(m, -\frac{L^\varphi - 2L^{2\varphi - d} + 2\chi}{\gamma_1}m\right)$ provided $d/2 > \varphi$, marginal if $d/2 = \varphi$ and attractive otherwise. This means that the only critical line which forms the basin of attraction for both fixed points is given only for $0 < \varphi < d/2$ and is locally defined by $g_1 = 0$. Here g_1 is the linear scaling field associated with the eigenvector $(m, 0)$.

The largest eigenvalue defines the critical exponent ν. In the trivial fixed point (m_1^*, λ_1^*), $\nu = 1/\varphi$ provided $d \geq \varphi$. If $d < \varphi$ than $\nu = \frac{1}{2\varphi - d}$. Here the eigenvalue $l_1 = L^\varphi > 1$ provided $2\varphi < d$; i.e. this fixed point is repulsive in the direction of the eigenvector $(m, 0)$ if and only if $2\varphi < d$. Although our results are rather general, let us consider the Flory's case as an example [9]. For $d \geq 5$ this trivial fixed point, in the Flory's case, is attractive in the direction of the eigenvector $(m, 0)$, marginal in dimension four and repulsive otherwise.

In the fixed point (m_2^*, λ_2^*), $\nu = \frac{1}{\varphi}$, provided $\beta > -log_L\left(\frac{1 + L^{\beta - d}}{2}\right)$. Back to the example we are considering here (Flory's case) [9]; for $d \geq 5$ this fixed point is repulsive, marginal in $d = 4$ and attractive otherwise.

We cannot explain, from this first order approach (the most probable event), logarithmic corrections to the end-to-end distance in the critical dimension. This is correctly explained, although heuristically, elsewhere [5].

Using the spin representation, we find the following.

a) For the trivial fixed point (m_1^*, λ_1^*).

$\alpha = 2 - d/\varphi$; $\beta = \frac{2(d-\varphi)}{\varphi}$; $\gamma = \frac{4\varphi - 3d}{\varphi}$; $\delta = \frac{2\varphi - d}{2d - 2\varphi}$; $\nu = \frac{1}{\varphi}$ and finally $\eta = 2 - 4\varphi + 3d$.

b) For the fixed point (m_2^*, λ_2^*).

$\alpha = 2 - d/\varphi$; $\beta = \frac{d - log_L(2 - L^{2\varphi - d})}{\varphi}$; $\gamma = \frac{2log_L(2 - L^{2\varphi - d}) - d}{\varphi}$; $\delta = \frac{Log_L(2 - L^{2\varphi - d})}{d - log_L(2 - L^{2\varphi - d})}$; $\nu = \frac{1}{\varphi}$ and finally $\eta = 2 + d - 2Log_L(2 - L^{2\varphi - d})$.

Besides, if we introduce critically the mass as was done in Brydges et al. [4] in $d = 4$ and $\varphi = 2$, the critical exponents look as follows.

$\alpha = 0$; $\beta = \frac{1}{2}$; $\gamma = 1$; $\delta = 3$; $\nu = \frac{1}{2}$ and finally $\eta = 0$.

On the other hand, we know that for the SUSY $\lambda\phi^4$, $\beta(\lambda') = \mu\frac{\partial\lambda'}{\partial\mu}$, where μ is a parameter with the dimensions of mass; namely μ is an arbitrary mass parameter.

Since we know the fixed points for the theory in random walk representation; these must be the zeros of $\beta(\lambda)$ in the SUSY $\lambda\phi^4$ representation. Using this criteria we obtain the following expression for $\beta(\lambda)$;

$$\beta(\lambda) = \frac{\gamma_2}{1 - L^\varphi}\lambda^2 - \frac{\gamma_1}{1 - L^\varphi}\lambda + m \tag{24}$$

up to a multiplicative constant.

An interesting pictorial interpretation of the renormalized group equation was suggested by S. Coleman [10]. The equation can be viewed as a flow of bacteria in a fluid streaming in a one dimensional channel. Here we provide a new interpretation of the velocity of the fluid at the point λ, $\beta(\lambda)$, in terms of stochastic events (function of conditional expectations of two-body interactions inside contracting G_1 cosets). For large lattice $\beta(\lambda)$ is independent of the lattice parameter L.

Concretely, $\beta(\lambda)$ (or the velocity of the fluid at the point λ) is written

in terms of one two-body and two two-body contributions to renormalized local times (stochastic approach) or mass (field theory approach). The first contribution is $O(\lambda)$ and the second, $O(\lambda^2)$.

Let us call $\beta'(\lambda) = \left(\frac{\partial\beta(\lambda)}{\partial\lambda}\right)_m$, then

$$\beta'(\lambda) = \frac{2\gamma_2\lambda - \gamma_1}{1 - L^\varphi} \tag{25}$$

In the trivial fixed point, $\beta'(\lambda_1^*) > 0$ (infrared stable), provided $\varphi > 0$ and $L \geq 2$; besides $\beta'(\lambda_1^*) < 0$ (ultraviolet stable), provided $\varphi < 0$ and $L \geq 2$. In the fixed point (m_2^*, λ_2^*), $\beta'(\lambda_2^*) \geq 0$ (infrared stable), provided $\varphi \geq 1/2\left[d + log_L\left(\frac{\gamma_1\chi}{2\gamma_2} + 1\right)\right]$, and $\beta'(\lambda_2^*) < 0$ (ultraviolet stable) otherwise. Here we define $d_H = log_L\left(\frac{\gamma_1\chi}{2\gamma_2} + 1\right)$ which is given in terms of the ratio for conditional expectations of two-body interactions which renormalizes to local time and two-body interactions. From this, the following estimates are obtained

    a) d=4; $\beta'(\lambda_2^*) \leq 0$, provided $d_H \geq 0$.
    b) d=3; $\beta'(\lambda_2^*) \leq 0$, provided $d_H \geq 1/3$.
    b) d=2; $\beta'(\lambda_2^*) \leq 0$, provided $d_H \geq 2/3$.
    d) d=1; $\beta'(\lambda_2^*) \leq 0$, provided $d_H \geq 1$.

5   Summary

Because of the equivalence between the polymer and SAW problems, functional integration methods were employed in the majority of theoretical approaches to these problems. It should, however, be remarked that the critical exponents for the SAW obtained by this method are only meaningful if the spatial dimensionality d is close to its formal value $d = 4$, and it is not yet clear how to get results for real space in this way. There is another method based on the search for a solution to the exact equation for the probability density of the end-to-end distance of the random walk [11]. By defining the

self- consistent field explicitly, the density could be found with the help of the Fokker-Planck equation. In this paper we provide another alternative view where the probability density, as a random function of the random walk, is proposed.

Discrete random walks approximate to diffusion processes and many of the continuous equations of mathematical physics can be obtained, under suitable limit conditions, from such walks. Besides we can look at this relation the other way around; that is, suppose that the movement of an elementary particle can be described as a random walk on the lattice, which represents the medium it traverses, and that the distance between two neighbouring vertices, though very small, is of a definite size; therefore the continuous equations can be considered as merely approximations which may not hold at very small distances. We show in this paper how the mathematical results are easily derived by standard methods. The main interest lies in the interpretation of the results.

In our approach the properties of the medium will be described by the lattice and the transition probabilities. We obtain m', the "mass" of the field as observed on this particular hierarchical lattice. The lattice is characterized by the ultrametric space used to label this.

We propose to obtain renormalized n-body interactions out of a set of stochastic diagrams with a fixed totally arbitrary topology.

Here we would like to stress that the search for a proper mathematical foundation of a physical theory does not mean only a concession to the quest for aesthetic beauty and clarity but is intended to meet an essential physical requirement. The mathematical control of the theory plays a crucial role to allow estimates on the proposed approximations and neglected terms.

Usually approximations must be introduced which often have the drawback that, although they can work well, they are uncontrolled: there is no small parameter which allows an estimate of the error.

Explicit mathematical formulae for all the parameters and remainders in the method can be provided. In sake of brevity we present these elsewhere

[5]. All of them are expressed in terms of conditional expectations of events inside contracting G_1 cosets.

Once a successful theoretical scheme has been found it is conceivable that it is possible to reformulate its structure in equivalent terms but in different forms in order to isolate, in the most convenient way, some of its aspects and eventually find the road to successive developments.

Let us remark that we are talking of a particle picture even when we deal with systems containing many particles or even field systems.

We hope our method and ideas may help in the proper understanding of the association of stochastic processes to the quantum states of a dynamical system; i.e. stochastic quantization.

Summarizing, in this paper we present an heuristic space-time renormalization-group map, on the space of probabilities, to study SUSY ϕ^4 in random walk representation, on a hierarchical metric space defined by a countable, abelian group G and an ultrametric δ. We present the Lévy process on Λ_n that correspond to the random walk representation of SUSY ϕ^4 which is a configurational measure model from the point of view of a stochastic processes. We apply the renormalization-group map on the random walk representation and work out explicitly the weakly SARW case for double intersecting paths which corresponds to SUSY ϕ^4, as an example. The generalization to SUSY ϕ^n, for any n, is straightforward. New conclusions are derived from our analysis.

Our result improves the field-theoretical approach [4] by obtaining an exact probabilistic formula for the flow of the interaction constant and the mass under the map.

6   Acknowledgments

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